Fact Sheet – Confidence Interval for an Unknown Population Proportion p (9.1)

In this section we learn to construct a confidence interval for an unknown population parameter (p) based upon the sample proportion $\left(\hat{p}\right)$.

Example: A survey of 125 COS students shows that 30 own an iPhone. Construct a 95% confidence interval for the proportion of all COS students that own an iPhone.

Example: In a medical study, 80 patients were given a new allergy medication and 24 experienced headaches as a side effect. Construct a 90% confidence interval for the proportion of all allergy patients who experience headaches as a side effect of taking the new medication.

Example: A survey of 100 COS students shows that 55 are female. Construct a 99% confidence interval for the proportion of all COS students that are female.

Conditions

To construct a confidence interval for an unknown population proportion, p, the following three conditions must be met.

- The sample is independently obtained using simple random sampling or through a randomized experiment.
- $n\hat{p}\left(1-\hat{p}\right)\geq 10$
- $n \leq 5\%$ of N or $20n \leq N$

Margin of Error   Lower Bound   Upper Bound

$$E = z \cdot \sqrt{\dfrac{\hat{p}\left(1-\hat{p}\right)}{n}}$$ $\hat{p} - E$ $\hat{p} + E$

Interpretation

We are ___ % sure that the true population proportion p is between __Lower Bound__ & __Upper Bound__ .

StatCrunch Steps

- Stat > Proportions > One Sample > with summary
- Enter the number of successes & number of observations. Next.
- Select the Confidence Level radio button and enter the level of confidence. Calculate.
- Round the lower bound and upper bound to 4 decimal places.

1) A survey of 125 COS students shows that 30 own an iPhone.

    a) Verify that the 3 conditions are met.

    b) Construct a 95% confidence interval for the proportion of all COS students that own an iPhone.
       Lower Limit: Upper Limit:

    c) Determine the margin of error, E.

2) In a medical study, 80 patients were given a new allergy medication and 24 experienced headaches as a side effect.

    a) Verify that the 3 conditions are met.

    b) Construct a 90% confidence interval for the proportion of all allergy patients who experience headaches as a side effect of taking the new medication. Determine the margin of error.
       Lower Limit: Upper Limit: E:

    c) Construct a 95% confidence interval for the proportion of all allergy patients who experience headaches as a side effect of taking the new medication. Determine the margin of error.
       Lower Limit: Upper Limit: E:

    d) Construct a 99% confidence interval for the proportion of all allergy patients who experience headaches as a side effect of taking the new medication. Determine the margin of error.
       Lower Limit: Upper Limit: E:

3) A survey of 100 COS students shows that 55 are female.

    a) Verify that the 3 conditions are met.

    b) Construct a 95% confidence interval for the proportion of all COS students who are female. Determine the margin of error.

       Lower Limit: Upper Limit: E:

    c) Change the sample proportion to 110 females out of 200 COS students and construct a 95% confidence interval for the proportion of all COS students who are female. Determine the margin of error.

       Lower Limit: Upper Limit: E:

    d) Change the sample proportion to 550 females out of 1000 COS students and construct a 95% confidence interval for the proportion of all COS students who are female. Determine the margin of error.

       Lower Limit: Upper Limit: E:

Fact Sheet – Appropriate Sample Size for Estimating p (9.1)

In this section we learn to determine how large of a sample is necessary to construct a confidence interval that meets certain criteria, including level of confidence and margin of error.

Example: I want to estimate the percentage of community college students who are female. How large of a sample must I take in order to be 95% confident that the sample proportion is within 5% of the true percentage?

Formula

$$n = \left(\frac{z}{E} \right)^2 \cdot p(1-p)$$

Information Needed

Proportion: Level of Confidence, Margin of Error, Estimate of Proportion

StatCrunch Steps

- Stat > Proportion Stats > One Sample > width/sample size
- Enter Confidence Level and Target Proportion (use 0.5 if unknown).
- Enter the Width, which is double the margin of error *E*. Press Compute to find sample size.

Examples - Sample Size for Estimating a Population Proportion

4) An ambitious health researcher wants to determine the proportion of American teenagers that are overweight. The researcher wants to be within 1% of the actual proportion. How large of a sample is necessary to be 95% confident that the sample proportion is within the researcher's desired margin of error?

5) A student wants to determine what percentage of college students smoke. How large of a sample should she take in order to be 90% confident that her sample proportion is off by no more than 4.5%?